

# Comment on “High-Temperature Series Analysis of the 2D Random-Bond Ising Ferromagnet”

In a recent Letter [1], Roder *et al* claimed that their high temperature series analysis of the two dimensional random bond Ising ferromagnet (RBIF) conclusively supported the prediction by Shalaev, Shankar, and Ludwig (SSL) [2]. The claim is based on the observation that their “ln-Pade” analysis of the magnetic susceptibility ( $\chi$ ) assuming

$$\chi \sim t^{-7/4} |\ln t|^p, \quad (1)$$

yields estimate of the logarithmic exponent  $p$  that is consistent with the predicted value of SSL,  $p = 7/8$ .

Being a perturbation theory, the theory of SSL is supposed to be more correct as the degree of disorder becomes smaller, that is, as the value of  $J_2/J_1$  becomes closer to one for the 2D RBIF. When the strength of disorder is extremely small, on the other hand, the critical behavior of the disordered Ising system must be almost indistinguishable from that of the pure system, so asymptotic expression of SSL, Eq.1 for  $\chi$ , is supposed to hold for extremely narrow scaling regime only. The remaining regime is unaffected by the presence of the disorder and maintains the scaling behavior of the pure system. Thus, in the context of the theory there generally exists a crossover from the critical behavior of the pure system to Eq.1 as  $t \rightarrow 0$ , which is reflected in the expression

$$\chi \sim t^{-7/4} [1 + g |\ln t|]^{\gamma'}, \quad (2)$$

with the value of the logarithmic exponent  $\gamma' = 7/8$ .

Note that Eq.2 reduces to the asymptotic form Eq.1 only when  $t$  is extremely small or the value of  $g$  is extremely large. The value of the  $g$  is supposed to increase smoothly with  $J_2/J_1$  from  $g = 0$  at  $J_2/J_1 = 1$ , but the theory is not able to determine  $g$  as a function of  $J_2/J_1$ . Since the crossover temperature is a priori unknown, Eq.2 instead of Eq.1 should be used for the analysis of series expansion or of Monte Carlo (MC) data.

The authors in the Letter simply assume that for  $5 \leq J_2/J_1 \leq 10$  the value of  $g$  is sufficiently large and that their series safely represents the asymptotic regime of SSL. Their estimate of  $p \simeq 7/8$  from ln-Pade analysis, however, indicates that the value of  $\gamma'$  be larger than the predicted value of SSL. This can be easily seen from Fig.2 of the Letter; for example for  $J_2/J_1 = 3$  where the prediction of SSL is supposed to be more correct than for  $J_2/J_1 \geq 5$ , their estimated value of  $p$  is just 0.3 whereas the value of  $\gamma'$  is supposed to be  $7/8$ .

The apparent plateau in the estimate of  $p$  (Fig.2 of the Letter) is surprising in light of the monotonically increasing critical exponent with  $J_2/J_1$  when analyzed assuming pure power law critical behavior (Fig.1): It is an elementary mathematical fact that  $\chi \sim t^{-\gamma}$  with  $\gamma(> 7/4)$

increasing with  $J_2/J_1$  is approximated with increasing value of  $p$  in Eq.1 rather than its fluctuating values. The monotonic increment of the critical exponent was clearly observed in the previous MC studies as well [3,4]. In fact MC study on the RBIF [4] and the series analysis yield completely agreeing estimates of the critical exponents. With this agreement the plateau in the value of  $p$  for the wide range of  $5 \leq J_2/J_1 \leq 10$  is rather spurious, probably resulting from subjective choice of the different orders of series terms for the different values of  $J_2/J_1$  in their analysis.

Their remarks about previous MC studies are also mainly incorrect. Especially, previous MC study [3] that supported varying critical exponent with the strength of random disorder is not based on finite size scaling analysis but on the careful measurements of the thermodynamic values of various physical quantities. It was shown that the thermodynamic data of correlation length and the magnetic susceptibility fit equally well to the scenario of varying critical exponent and to the predictions of SSL. However, the data of the specific heat at least for strongly disordered case was manifestly inconsistent with the double logarithmic behavior predicted by SSL.

To sum up: Estimate of the logarithmic exponent is very sensitive depending on which critical singularity between Eq.1 and Eq.2 is used for analysis, and it cannot be justified that the value of  $g$  is so large for  $J_2/J_1 \geq 5$  that Eq.1 is a valid one for their analysis. In general,  $p$  must be regarded as a lower bound of  $\gamma'$  so that  $p \simeq 7/8$  actually indicates  $\gamma' \gtrsim 7/8$ . Furthermore, the plateau in the value of  $p$  seems to be spurious. We thus conclude that their claim is groundless. In parallel with MC study, it would be interesting to see the series analysis of the specific heat for strongly disordered case.

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